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AN

INTRODUCTION

TO

PLAIN TRIGONOMETRY,

WITH ITS APPLICATION

TO

HEIGHTS and *DISTANCES.*

Containing an Explanation of the three Varieties of right angled Triangles, and the four Caſes of Oblique, together with a Variety of Queſtions interſperſed by way of Exerciſe.

By RICHARD COCKREL,

Teacher of the Free School, at LARTINGTON.

* * *

> *Perveniri ad ſumma, niſi ex principiis, non poteſt.* Quint.

DARLINGTON:

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MDCCXCII.

ABBREVIATIONS.

+ ſignifies Addition, ſo 2 + 4 means that 2 and 4 are to be added together.

— denotes Subtraction, ſo 4—2 ſignifies that 2 is to be taken from 4.

= denotes Equality, ſo 4 = 3 + 1 ſignifies that 4 is equal 3 when added to 1.

∠ ſignifies any Angle.

THE PREFACE.

THE *Number of Publications already extant, treating on* TRIGONOMETRY, *I am well perſuaded occupy no inconſiderable Part of the* Bookſeller's *Shelves. But as Improvement does not always keep Pace with the Time and Labor beſtowed on a Subject, ſo the Treatiſes already abroad on this Science, do not exclude any Neceſſity for further Elucidation.*

The two following ſubſtantial Reaſons, are what I lay before the Public as an Apology for ſending this Work *abroad into the World.* I. *That the* Books *for the moſt part which contain any Thing of* Plain Trigonometry, *are Books which treat of one or more of the other mathematical Sciences, as* Menſuration, Navigation, Aſtronomy, &c. *and thereby the Price of the Books becomes* 5, 6 *or perhaps* 7s. 6d. *Purchaſe. Now I have by treating of this Branch ſingly, reduced the Purchaſe, ſeveral Shillings—an Object you know to thoſe in poor Circumſtances! And* II. *The Way in which I have treated this Subject, is entirely new, I mean as to the Plan. And as far as I ſee,* *(but to be ſure a Father is apt to judge partially of his own Child) it is ſo plain and eaſy that to be comprehended, it need only to be read. I have taught by this Method for nearly* 6 *Years, and with ſuch Succeſs, that I am induced to lay it open to the World, in hopes that it may facilitate the Acquiſition of ſo uſeful a Science.*

I have in the firſt Place treated on Practical Geometry, *but have uſed only ſuch Problems as the Work requires; looking upon any other as ſuperfluous.*

You have, in the next Place, given a Compendium of Speculative Geometry, *in Order to prepare the Learner for a right Underſtanding of the Demonſtrations of the ſeveral Rules which are to be found in the Work. Some of the Propoſitions I have ſelected from other Authors.*

As to the Plan purſued in ſolving the 3 Varieties *&c. of* Trigonometry, *I ſhall not ſpeak of here, as a few Obſervations, by Way of an Advertiſement to the Learner, are laid down on* *the Subject and placed before the* 1*ſt.* Variety, *and therefore ſhall only ſay, that I have rendered the Application of* Trigonometry *to* Heights and Diſtances, *as uſeful as poſſible, by inſerting various Kinds of Examples, ſeveral of which are borrowed from* Dr. HUTTON's *excellent Treatiſe on* Menſuration.

In ſhort, I have endeavoured throughout the whole, to be as plain and conciſe as poſſible, and indeed too much Pains and Labor cannot be beſtowed (to make any Improvement) on the Science of Trigonometry, *it is of ſuch general Utility. It may with Propriety be termed the Baſis of mathematical Learning. The Surveyor in meaſuring and laying out his Lands knows the Utility of it. The Mariner daily and hourly experiences convincing Proofs of its ſalutary Effects. But doth it reſt here? No—ſee it mount up to the Skies to ſupport the Aſtronomer in his lofty Reſearches, leads him on to new Diſcoveries, finds the Magnitude, Diſtance, Motion, Eclipſes &c. of the Heavenly Bodies, and explains the Cauſe of the Diverſity of Color in the Rainbow, to the natural Philo-*

ſopher. Such are the Powers of Trigonometry. *It is to this Science that we owe many valuable Diſcoveries—Diſcoveries which would perhaps for ever have been numbered with the Arcana of Nature.*

Therefore, conſidering its general Utility, I would adviſe every one who means to ſtudy to any great Length, to get acquainted with the Principles of it, for I am well aſſured, that no one who is ignorant of them, can make any great Progreſs in the mixed Mathematics.

Oſman-Flat,
Oct. 22*d.* 1792.

RICHARD COCKREL.

PRACTICAL GEOMETRY.

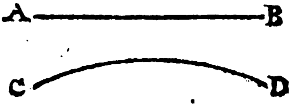
CHAP. I.

SECTION I.

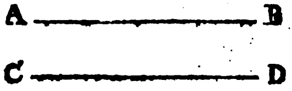
DEFINITIONS.

1. GEOMETRY is a Science which treats of extended Quantities: Extention is 1*ſt*. Length, 2*dly*, Length and Breadth, and 3*dly*, Length, Breadth and Thickneſs.

2. A Point is indiviſible and therefore has no Parts.

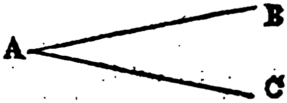


3. A Line is generated by the Motion of a Point whether it be ſtraight as A B, or in a curve as C D.

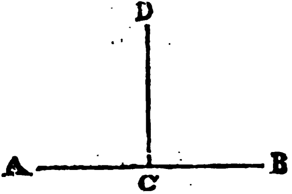


4. Parrallel Lines are ſuch as are equidiſtant in all their Parts, from each other, as A B, and C D.

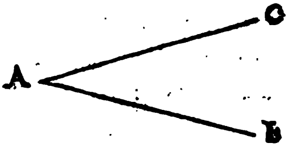
5. Two right Lines as A B, and A C, do, when they meet, make what is called an Angle, as A.



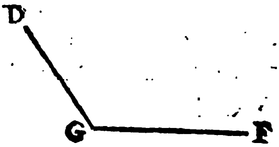
6. When one right Line falls on another right Line, and makes the Angles on each ſide equal, it is ſaid to be perpendicular to the Line on which it falls; and the Angles are called right Angles. Thus: D C is perpendicular to A B; and the Angle A C D equal the Angle D C B, is a right Angle.



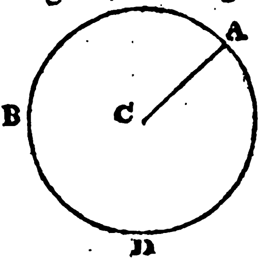
7. An acute Angle is leſs than a right Angle, as the Angle C A B.



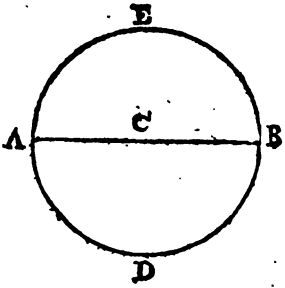
8. An obtuſe Angle is greater than a right Angle, as D G F.



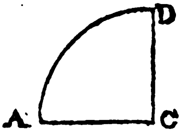
9. A Circle is a regular Figure, and is generated by the Revolution of the Line C A, (round the Centre C) called its Radius; the Line B D A, deſcribed by the Point A, in this revolution, is called the Circumference or Periphery of the Circle.



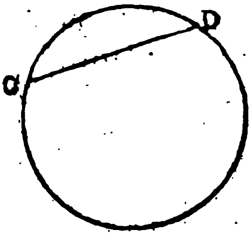
10. A Line paſſing through the Centre of a Circle, and terminated by the Circumference, is called a Diameter, as A B. It divides the Circle into two equal Parts called Semicircles, as A E B, and A D B.



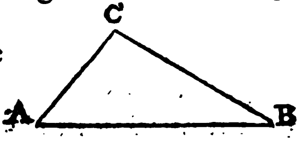
11. A Quadrant is half a Semicircle cut off by a Line perpendicular to the Diameter, as A C D.



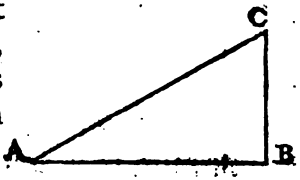
12. A Chord Line is ſuch a Line as cuts the Circle into 2 unequal parts, and is leſs than the Diameter, as C D.



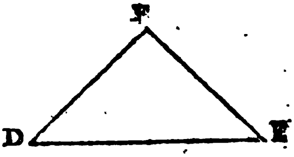
13. A Triangle is a Figure bounded by three right Lines, and conſequently has three Angles, as A B C



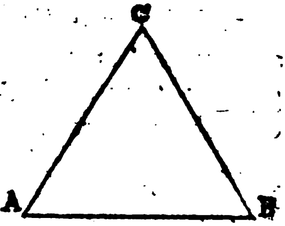
14. A right angled Triangle is ſuch as has one of its Angles a right one: or, in other words, when two of its Sides fall perpendicular to each other, as A B C.



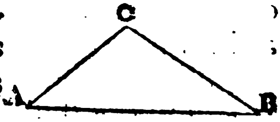
15. An Iſoſcales Triangle is ſuch as has two of its Sides equal, as D F=E F.



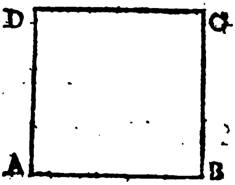
16. An equilateral Triangle hath all its Sides and Angles equal, as A C=B C=A B.



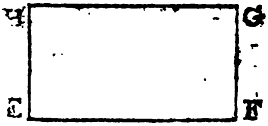
17. A Scalenous Triangle hath all its Sides and Angles unequal, as A C, B C, A B.



18. A Square has 4 equal Sides and all perpendicular to each other, as A B C D.



19. A Rectangle or Parallelogram is a four ſided Figure whoſe oppoſite Sides are equal, and as in the Square, the Sides are perpendicular to each other, as E F G H.



PROBLEM I.

To biſect a given right Line, or divide it into two equal Parts.

1. With

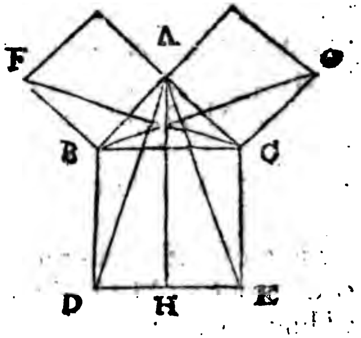
COROLLARY.

Hence Triangles upon the ſame Baſe, and between the ſame Parallels are equal, becauſe they are Halfs of Parallelograms deſcribed on the ſame Baſe, and between the ſame Parallels.

PROPOSITION II.

In every right angled Triangle, the Square of the Hypothenuſe (or Side ſubtending the right Angle,) is equal to the Sum of the Squares of the other two Sides.

DEMONSTRATION.



Draw the Line A H, Parallel to B D, and join A D and AE, the Angle D B C, is equal to the Angle F B A, and add the Angle A B C to both, then is the Angle A B D equal to the Angle F B C, moreover A B is equal to F B, and B D equal to B C, by the Definition of a

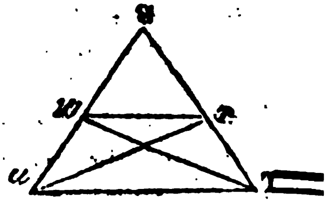
Square; therefore is the Triangle A B D, equal to the Triangle F B C, by Prop. 6. But the Parallelogram B H, is equal to twice the Triangle A B D, and the Parallelogram C H, is equal to twice the Triangle A C E, by Corol. Prop. 10. alſo the Square A F, is double the Triangle F B C, and the Square A G, double the Triangle G C D. But the Triangle F B C, is equal to the Triangle A B D. And the Triangle G C B, is equal to the Triangle A C E; therefore the Square F A, is equal to the Rectangle B H, and A G equal to C H, and by conſequence the Squares A F and A G, are equal to the Square B D E C. Q E D.

PROPOSITION 12.

If a Line be drawn parallel to any Side of a given Triangle, it will cut the other two Sides proportionally.

DEMONSTRATION.

Draw W X Parallel to U T, to cut the Sides S U and S T in W, X; then S W: S U:: S X: S T; draw W T and U X, making the Triangle



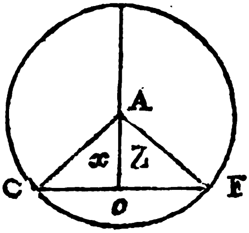
angles U W X and T X W; therefore it will be, as S W X: W U X: : S W: W U, and, as S W X: W U X (the Triangles U W X and T X W being equal) : : S X: X T; then throw away the two firſt Terms S W X, W U X, and it will be, as S W: W U: : S X: X T. Q. E D.

PROPOSITION 13.

If in a Circle, a right Line drawn through the Centre biſects another, not drawn through the Centre, it will cut it perpendicularly, and if it cut perpendicularly it will biſect it.

DEMONSTRATION.

Part 1. Fom the Centre A, let there be drawn A C, A F the Triangles X and Z are equilateral to each other. For CO, F O are by the Hypotheſis equal, and A C, A F are ſo, becauſe drawn from the Centre; while A O is common to both. Therefore the Angles A O C, A O F are equal by Prop. 6. Therefore right ones, by Prop. 1. which was the firſt part.



2. Becauſe by the Hypotheſis A O C, A O F are right Angles; A C Square ſhall be equal to the Squares of A O, O C together, by Prop 11; and A F Square, equal to the Squares of A O, O F together. Seeing therefore the Squares of A C, A F are equal, the Squares of A O, O C together will be alſo equal to the Squares of A O, O F together. Wherefore taking away the common Square A O, the Squares of O C, O F remain equal. And therefore the right Lines O C, O F are equal. Which was the other part.

TRIGONOMETRY.

TRIGONOMETRY.

SECTION 3.

DEFINITIONS.

Trigonometry *is that Part of the mixed Mathematics, that teaches us to calculate the Meaſures and Proportions of the ſeveral Parts of Triangles. The Parts of a plain Triangle are ſix: three Sides and three Angles. Theſe are therefore the ſubject of* Plain Trigonometry. *And in all our Enquiries three Things (either all the Sides, or Sides and Angles,) are given to find a fourth, (Side or Angle.)*

1. EVERY Circle whether great or ſmall is conceived to be divided into 360 equal Parts called Degrees, theſe Degrees are ſubdivided again into 60 equal Parts called Minutes, theſe again into Seconds &c. *See Note to Prob. 4. Sect. 1.*

2. A plain Triangle is projected on a plain or flat Superficies, and therefore its Sides are right Lines.

3. A plain Triangle is either right angled or oblique angled.

4. A right Angle is ſuch as meaſures juſt 90 Degrees, therefore a Triangle which has one of its Angles equal to 90 Degrees is called a right angled Triangle.

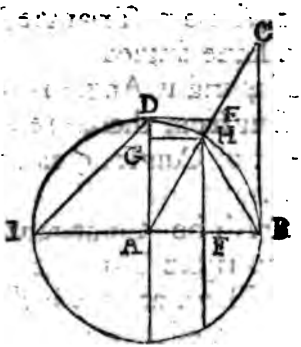
5. An oblique angled Triangle, is ſuch as has no right angle.

6. In right lined Trigonometry the Sum of the Angles of any Triangle, is always a Semicircle or 180 Degs. (Prop. 4. Sect. 2.) therefore in a right angled Triangle, one Angle (i. e. the right one) is always underſtood to be given; and if another Angle is given, in courſe, by ſubtracting it from 90 Degs. or adding it to the right Angle, and ſubtracting their Sums from 180 Degs. it will leave the third Angle.

7. In a right angled Triangle, the Side oppoſite to the right Angle, is called the Hypothenuſe and the two Sides containing the right Angle, are called the Baſe and Perpendicular.

8. The Complement of any Number of Degs. is what they want of 90 Degs. as H D is the complement of B H.

9. The



9. The Supplement of any Number of Degs. is what they want of 180. as H D I is ſupplement of B H.

10. The *Sine* of any Arch, is a Line drawn from one End of it perpendicular to a Diameter that paſſes through the other End, as H F is the Sine of H B.

11. The *Co-ſine* of any Arch, is the Sine of its Complement as H G is the Co-ſine of the Arch H B, or the Sine of its Complement H D.

12. The verſed Sine of any Number of Degs. is that Space which is contained between the Sine and Circumference, meaſured upon a Line paſſing through the Centre, as F B is the verſed Sine of H B.

13. The Coverſed Sine of any Number of Degs. is the verſed Sine of the Complement of thoſe Degrees as G D, is the coverſed Sine of B H, or verſed Sine of the Complement of thoſe Degrees,

14. The

II. In the ſecond *the Baſe is ſuppoſed Radius, and then the Perpendicular becomes Tangent, and Hypothenuſe* Secant.

III. In the third *the Perpendicular is conſidered as Radius and the Hypothenuſe will be Secant and Baſe Tangent.*

In order to illuſtrate the Proportions and aſſiſt the Learner to diſcover the true Relation, one Part of the Triangle has to another, three Plates are added, adapted to the 3 Varieties. In theſe the Triangle is diſſected and each Part, whether Side or Angle, placed oppoſite its correſponding Part, with the proper term affixed to both. Thus the Learner needs only turn to the Plate and conſider what Parts he knows, and what he wiſhes to diſcover, and he will inſtantly ſee how his proportion is to be formed. But more of this when we come to the Examples.

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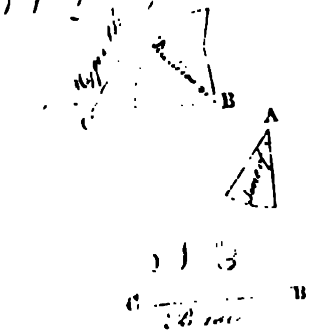

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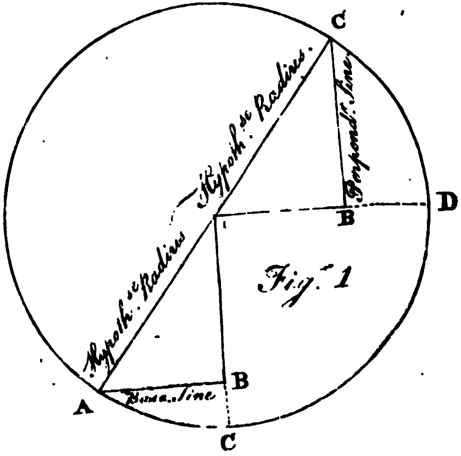


Fig.e 1

Right angled Triangles.

VARIETY 1.

Hypothenuſe Radius.

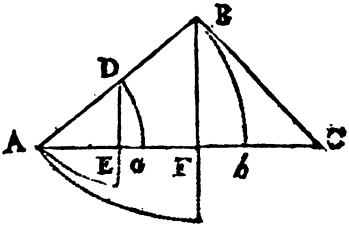
IF the Hypothenuſe be made Radius, each Leg will be the Sine of its oppoſite Angle. late 1. Fig. 1. Where it is evident (from efinition 7) that A B is the Sine of the Arch C, and B C the Sine of the Arch C D.

ule * *As any one Side, is to the Sine of its oppoſite Angle, ſo is any other Side, to the Sine of its oppoſite Angle; and vice verſa.*

---

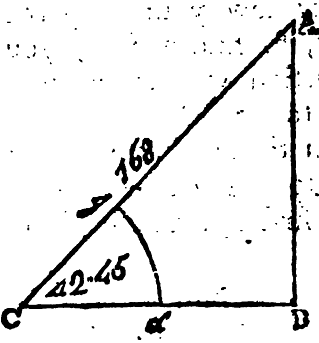
* DEMONSTRATION.

Let A B C be a Tringle. From D and B, t fall the Perpendicular E and B F. Now D a nd B b being deſcribed ith the Radii A D and B, ſhew that D E and F are Sines of their opoſite Angles (by Def. 7.) nd ſo of A F and A E. hen per ſimilar Tringles (Prop. 12. Sect. 2.)



as A B: B F:: A D: D E
and
as A B: A F:: A D: A E &c. Q.E D.

In the right angled Triangle A B C are given, the Hypothenuſe A C 168, and the Angle C 42°45′ to find the Perpendicular.



GEOMETRICALLY.

Firſt draw the Line C B at Pleaſure, with a Chord of 60 Degs, in your Compaſſes, and one Foot in C, ſweep the Arch d g, and lay off the Chord, 42°45′, through g draw C A, which meaſure 168; from A, let fall the Perpendicular A B, then is the Triangle conſtructed geometrically, the Perpendicular meaſuring on a Scale of equal Parts 114, and Baſe BC 115·1.

Logarithmetically

LOGARITHMETICALLY.

You have the Hypothenuſe and all the Angles given, but neither the Perpendicular nor Baſe. Now turn to Plate 1, and you will ſoon diſcover how your Proportion will run. Remember, to find a Side, you muſt begin your Proportion with an Angle, which muſt be oppoſite to a given Side; therefore ſeek in the Plate for the Hypothenuſe given, and you will find it to be the Sine of 90 Degs. which muſt be the firſt Term in the Proportion, thus;

As Radius, or Sine of 90°	10·000000
Is to the Hypothenuſe 168	2·225309

Next look for the other given Angle C, and you will find the Perpendicular to be the Sine of it, which will finiſh the Proportion.

So is the Sine of ∠ C 42° 45′	9·831742
To the Perpend. 114	2·057051

Again, to find the Baſe, you may uſe the former Part of the laſt Proportion, if you pleaſe, or the Latter either, it matters not which;

which ‡; then look for the required Side AB which you will find No. 3. with the Angle A oppoſite to it, therefore your ſecond Proportion will be,

As Radius, or Sine of 90°		10·000000
Is to the Hypoth.	168	2·225309
So is Sine of the ∠ A	47°,,15′	9·865887
To the Baſe	124	2·091196

or

As the Sine of the ∠ C	42°,,45′	9·831742
Is to the Perpend.	114	2·057051
So is the Sine of the ∠ A	47°,,15′	9·865887
To the Baſe	124	2·091196

Queſtion 2. Given in the right angled Triangle ABC, the Perpendicular BA 114 and the Angle C 42°,,45′; required the Hypothenuſe and Baſe.

Geometrically.

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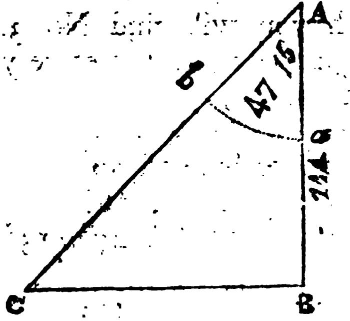
‡ That taking either Part of the Proportion makes no Alteration in the Solution will appear evident, when it is conſidered, that as any given Side is to the Sine of its oppoſite Angle, ſo is any other Side to the Sine of its oppoſite Angle, and *vice verſa*.



GEOMETRICALLY.

In the firſt place draw the Line A B, and ſet off 114 thereon, then raiſe B C perpendicular thereunto; next with the Chord of 60° in your Compaſſes, ſweep the Arch a b and lay off 47° 15′, through b produce the Line A C, which completes the Triangle.

N. B. If A C and C B be applied to the Scale of equal Parts, it will give their reſpective Lengths.

LOGARITHMETICALLY.

You have the perpendicular and Angles given to find the Hypothenuſe and Baſe. Turn to Plate 1, and ſeek for the Perpendicular, which you will find (No. 1) to be the Sine of its oppoſite Angle C, then in the firſt place it will be,

As Sine of the ∠ C 42° ,, 45′, 9·831742
Is to the Perpend. 114 2·057051

Look next for the other Angle A, and you will obſerve (No. 3.) the Baſe to be the Sine of it, which will finiſh the Proportion.

So is Sine of the ∠ A 47° ,, 15′ 9·865887

To the Baſe 124 2·091196.

Again, to find the Hypothenuſe, you may make uſe either of the former or latter Part of the laſt Proportion, and looking in the Plate for the required Side C A, you will find it (No. 3.) with the Radius oppoſite to it, conſequently you proceed, thus:

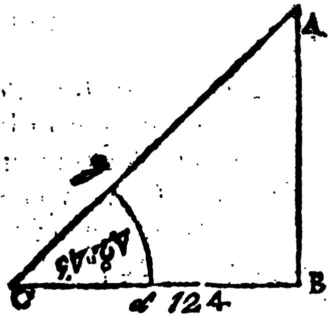
As Sine ∠ C 42° ,, 45′ 9·831742
Is to the Perpend. 114 2·057051
So is Radius 90° 10·000000

To the Hypoth. 168 2·225309

or

As Sine of the ∠A 47°,, 15′	9·865887
Is to the Baſe 124	2·091196
So is Radius 90	10·000000
To the Hypoth.	2·225309

Queſtion 3. Given in the right angled Triangle A B C, the Baſe B C 124 and ∠C 42°,, 45′ required the Hypothenuſe and Perpendicular.



GEOMETRICALLY.

Draw the Baſe B C, and lay off thereon 124 from a Scale of equal Parts; next on the Point B, raiſe the Perpendicular B A, then with the Sweep of 60° in your Compaſſes deſcribe the

Arch d g, and lay off thereon 42° ,, 45' from the Line of Chords from d to g, join C A produced through g which completes the Triangle.

LOGARITHMETICALLY.

You have the Baſe and Angle C given, but neither the Hypothenuſe nor Perpendicular, therefore turn to Plate 1, and find which Angle the Baſe is the Sine of, which you will find (No. 3) to be the Angle A 47° ,, 15'. The two firſt Terms of your Propoſition will be as follows.

As Sine of the ∠ A	47 ,, 15'	9·865887
Is to the Baſe	124	2·091196

Look next for the Angle C, which you will find (No. 1.) oppoſite to the Perpendicular; therefore,

So is Sine of the ∠ C	42° ,, 45'	9·831742
To the Perpend.	114	2.057051

Again to find the Hypothenuſe;

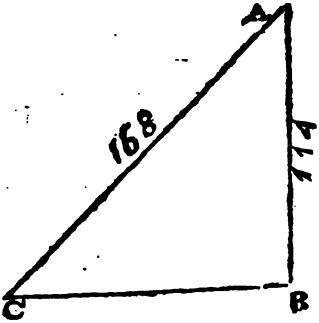
As Sine of the ∠ C	42° ,, 45'	9·831742
Is to the Perpend.	114	2·057051
So is Radius	90	10·000000
To the Hypoth.	168	2·225309

As

or

As Sine of the ∠A 47° 15'	9·865887
Is to the Baſe 124	2·091196
So is Radius 90	10·000000
To the Hypoth. 168	2·225309

Queſtion 4. Given in the right angled Triangle A B C, the Hypothenuſe A C 168 and Perpendicular B A 114 to find the Angles and Baſe.



GEOMETRICALLY.

Draw the Perpendicular A B and lay off thereon 114, then from the Point B, draw B C perpendicular to A B; next take the Length of the Hypothenuſe A C in your Compaſſes, and one Foot in A, as a Centre, with the other cut the Line B C, as in C, then a Line drawn from A to C completes the Triangle.

N. B. With the Chord of 60° in your Compaſſes and one Foot in C, deſcribe the Arch d g, which meaſured in your Compaſſes and applied to the Line of Chords will give its Length.

LOGARITHMETICALLY.

You have given here, the Perpendicular and Hypothenuſe, but neither the Baſe nor Angles. Turn to Plate 1, and ſee what Angle the Hypothenuſe is oppoſite to, which you will find (No. 2.) to be Radius 90° therefore in the firſt place it will be,

As the Hypoth. A C 168	2·225309
Is to Radius 90	10·000000

Look next for the Perpendicular which you will find (No. 1.) the Sine of the Angle C, therefore you proceed, thus:

So

So is the Perpend. 114	2.057051
To Sine of the ∠ C 42°,, 45′	9·831742

Again to find the Baſe, you muſt begin with an Angle, as directed in a former Page; therefore look in the Plate for the Baſe, and you will find it (No. 3.) to be the Sine of the Angle A, your Proportion then will run, thus:

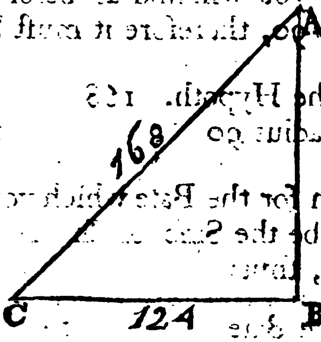
As Radius 90	10·000000
Is to the Hypoth. 168	2·225309
So is Sine of the ∠ A 47°,, 15′	9·865887
To the Baſe 124	2·091196

or

As Sine of the ∠ C 42°,, 45′	9·831742
Is to the Perpend. 114	2·057051
So is Sine of the ∠ A 47,, 15′	9·865887
To the Baſe 124	2·091196

Queſtion 5. In the right angled Triangle A B C, are given the Hypothenuſe 168 and Baſe 124 to find the Angles and Perpendicular.

Geometrically



GEOMETRICALLY.

Firſt draw the Line B C and lay off thereon 124; on B erect a Perpendicular, and with an Opening of the Compaſſes equal to the Length of the Hypothenuſe, ſet one Foot in C and with the other cut B A in A, a Line drawn from C to A completes the Triangle.

N. B. The Angles C and A may be found as directed in the laſt Queſtion.

LOGARITHMETICALLY.

In this Queſtion you have given the Hypothenuſe and Baſe, but neither the Perpendicular nor Angles. Turn to Plate 1, and ſee what Angle the Hypothenuſe is connected with, which you will find as before (No. 2.) to be Radius 90, therefore it muſt be,

As to the Hypoth.	168	2·225309
Is to Radius 90		10·000000

Look then for the Baſe which you will find (No. 1.) to be the Sine of the Angle A, ſo you proceed, thus;

So is the Baſe	124	2·091196
To Sine of the ∠ A 47°,, 15′		9·865887

Again, to find the Perpendicular.

As Radius, 90°		10·000000
Is to the Hypoth.	168	2·225309
So is Sine of the ∠ C 42,,45		9·831742
To the Perpend.	114	2·057051

or

As Sine of the ∠ A 47°,, 15′		9·865887
Is to the Baſe	124	2·091196
So is Sine of the ∠ C 42°,,45′		9·831742
To the Perpend.	114	2·057051

Queſtions

Queſtions to exerciſe the firſt Variety.

1. In the right angled Triangle A B C are given the Hypothenuſe A C 100, and the Angle A C B 52° ,, 10', required the Baſe and Perpendicular?

Ans. Baſe 77·9 and Perpend. 62·71.

2. Given, in the right angled Triangle A D G, the Perpendicular A D 75 and the Angle ∠ D 33° ,, 19' to find the Baſe and Hypothenuſe.

Ans. Baſe 49·3 and Hypoth. 89·7

3. Suppoſe in the right angled Triangle X Y Z, the Baſe X Y is 505, and the Angle Y 40° what are the Perpendicular and Hypothenuſe.

Ans. Perpend. 423·8 and Hypoth. 659·2

4. I find in the Triangle B H S, that the Baſe B H is 87 and Hypothenuſe B S 108 quere the Perpendicular and Angles.

Ans. Perp. 63·99, Angle B 30° ,, 20' & S 53° ,, 40'

5. There is a Triangle whoſe Hypothenuſe is 78 and Perpendicular 60. What are the Angles and Baſe?

Ans. the Angle oppoſite the Perp. 50°„17′ }
and Baſe 49‧84. }

6. In the Triangle F C D, are given the Hypothenuſe F D 157, and the Angle DFC 33°„45′ quere the Baſe and Perpendicular?

Ans. the Baſe 130 and Perpend. 87‧2

* * *

VARIETY 2.

Baſe Radius.

IF the Baſe be made Radius the Perpendicular becomes Tangent and Hypothenuſe Secant, See Plate 2, Fig. 2; where it is evident (from Def. 14 and 16) that as the Baſe A B is made Radius, B C is Tangent, and the Hypothenuſe A C Secant.

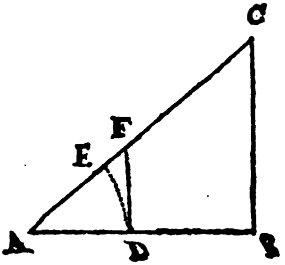
RULE*

As the Radius
Is to the Baſe
So is Tang. of the Angle
oppoſite to the Perpend
To the Perpend.
and
As the Radius
Is to the Baſe
So is the Secant of the ſame ∠
To the Hypothenuſe.
And the contrary. In

---

* DEMONSTRATION.

With the Centre A, and any Radius A D, deſcribe an Arch D E, and erect the Perpendicular D F, which, it is evident, will be the Tangent, and A F the Secant of the Arch D E, or Angle A to the Radius A D. And in the Similar Triangles *A D F*, ABC, it will be A D: *A B* : : *D F*: BC : : AF: AC. *Q E D.*



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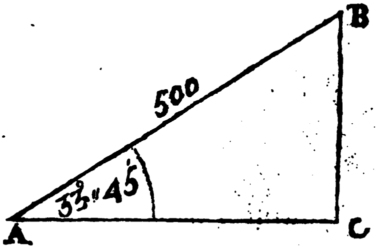
ueſtion 1. In the Triangle A B C are given Hypothenuſe A C 500 and the Angle A ″ 45′, to find the Perpendicular and Baſe.



GEOMETRICALLY.

s the Geometrical Conſtruction in this the following Variety is much the ſame he former, I ſhall therefore ſay no more hat Head.

GARITHMETICALLY.

ou have the Hypothenuſe and all the gles given, but neither the Baſe nor Per- dicular. Now turn to Plate 2, and you ſoon diſcover how your Proportion will , for firſt ſeek for the Hypothenuſe given, you will find it to be (No. 2.) Secant of Angle A, therefore you begin, thus:

As Secant A ∠	33″ 45	10·080153
Is to the Hypoth.	500	2·698970

As the Baſe is made Radius, and you have to find the Baſe, the other Part of the Proportion will be,

So is Radius 90	10.000000
To the Baſe 415·8	2·618817

Next you have to find the Perpendicular; look into the Plate and you will find it to be Tangent of the Angle A, therefore to finiſh your Proportion, proceed by the latter Part of the laſt as follows,

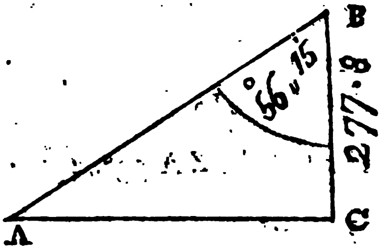
As Radius 90	10·000000
Is to the Baſe 415·8	2·618817
So is the Tang. of ∠ A 33,,45	9·824892
To the Perpend. 277·8	2·443709

Or which will anſwer the ſame Purpoſe,

As Secant of ∠ A 33°,,45'	10·080153
Is to the Hypoth. 500	2·698970
So is Tang. of ∠ A 33,,45	9·824892
To the Perpend. 277·8	2·443709

N. B. The Side made Radius generally comes firſt in the Proportion, if it is given.

Queſtion 2. In the right angled Triangle ABC, are given, BC the Perpendicular 277·8 and the Angle A 33° 45′ to find the Baſe and Hypothenuſe.



LOGARITHMETICALLY.

Here you have given the Perpendicular and Angle to find the Baſe and Hypothenuſe. Firſt look into Plate 2, and you will find as before, that the Perpendicular is Tangent of the Angle A, therefore,

As Tang. ∠ A 33″45	9·824892
Is to the Perpend. 277·8	2·443709

The other Part of the Proportion will be as before.

So

So is Radius 90	10·000000
To the Baſe 415·8	2·618817

Now by looking into Plate 2, you will find that the Hypothenuſe is Secant of the Angle A; therefore you may either ſay,

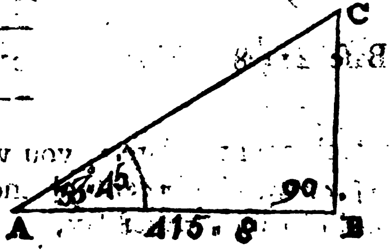
As Radius, 90	10·000000
Is to the Baſe 415·8	2·618817
So is Secant of the ∠A 33° 45′	10·080153
To the Hypoth. 500	2·698970

or

As Tang. of the ∠A 33° 45′	9·824892
Is to the Perpend. 277·8	2·443709
So is Secant of the ∠A 33° 45′	10·080153
To the Hypoth. 500	2·698970

Queſtion 3. In the right angled Triangle A B C are given the Baſe A B 415·8 and Angle A 33° 45′ to find the Perpendicular and Hypothenuſe.

Logarithmetically



LOGARITHMETICALLY.

Here you have the Baſe and Angles given to find the Perpendicular and Hypothenuſe. In the firſt Place, becauſe the Baſe is made Radius and you have it given, you muſt bring it in firſt, in the Proportion, agreeable to what is obſerved by way of Note, Queſtion 1. But a Side is required, therefore you muſt begin with an Angle, which Angle will be the Radius 90 by the Plate.

As Radius 90		10·000000
Is to the Baſe	415·8	2·618817

Now to find the Perpendicular, look into the Plate and you will find it Tangent of the Angle, A as before, therefore

So

So is Tang. of the ∠ A	33° 45'	9·824892
To the Perpend.	277.8	2·443709

Again to find the Hypothenuſe.

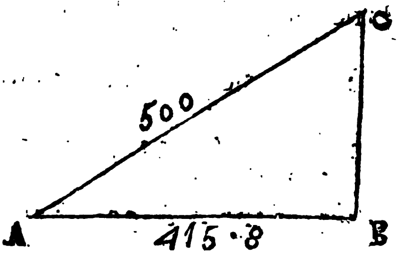
As Tang. of the ∠ A	33° 45'	9·824892
Is to the Perpend.	277·8	2·443709
So is Secant of the ∠ A	33° 45'	10·080153
To the Hypoth.	500	2·698970

or

As Radius 90		10·000000
Is to the Baſe	415·8	2.618817
So is Secant of the ∠ A	33° 45'	10·080153
To the Hypoth.	500	2·698970

Queſtion 4. In the right angled Triangle A B C, are given, the Baſe A B 415·8 and Hypothenuſe A C 500, to find the Angles and Perpendicular.

Logarithmetically.



LOGARITHMETICALLY.

Both the Baſe and Hypothenuſe are here given, but neither the Angles nor Perpendicular. Now as an Angle is to be found, a Side muſt come firſt in Proportion, which muſt be the Baſe, becauſe it is made Radius.

As the Baſe	415·8	2·618817
Is to Radius	90	10·000000

The Hypothenuſe will be the Secant of the ∠ A, as before, therefore

So is the Hypoth.	500	2·698970
To Secant of the ∠ A	33° 45′	10·080153

N. B. This Angle ſubtracted from 90° will give the other Angle, by Definition 5.

For the Perpendicular, it may either be,

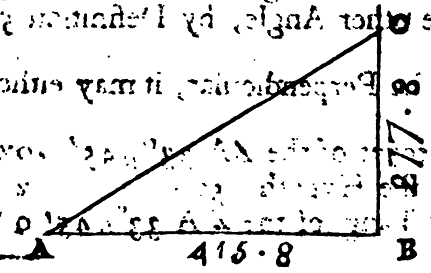
As Secant of the ∠A 33° ,, 45′	10·080153
Is to the Hypoth. 500	2·698970
So is Tang. of the ∠ A 33° ,, 45′	9·824892
To the Perpend. 277·8	2·443709

or

As Radius 90	10·000000
Is to the Baſe 415·8	2·618817
So is Tang. of the ∠ A 33° ,, 45′	9·824892
To the Perpend. 277·8	2·443709

Queſtion 5. In the right angled Triangle A B C, there are given the Baſe A B 415·8, and Perpendicular B C 277·8 to find the Angles and Hypothenuſe.

Logarithmetically.



LOGARITHMETICALLY.

The Baſe and Perpendicular are here given but neither the Angles nor Hypothenuſe. As an Angle is to be found you muſt begin with a Side, and that Side muſt be the Baſe, therefore,

As the Baſe	415·8	2·618817
Is to Radius	90	10.000000
So is the Perpend.	277·8	2·443799
To the Tang. of the ∠ A	33,45	9·824982

N. B. This Angle ſubtracted from 90 Degrees will give the other, Definition 6.

For the Hypothenuſe you muſt begin with an Angle, therefore it may be,

As Tang. of the ∠ A 33°,,45'	9·824892
Is to the Perpend. 277·8	2·443709
So is Secant of the ∠ A 33°,,45'	10·080153
To the Hypoth. 500	2·698970

or

As Radius 90	10·000000
Is to the Baſe 415·8	2·618817
So is Secant of the ∠ A 33°,,45'	10·080153
To the Hypoth. 500	2·698970

——o——o——

Queſtions to exerciſe the Second Variety.

1. In the right angled Triangle A B C are given the Hypothenuſe A C 817, and the Angle A 48°,,45' to find the Baſe and Perpendicular.

Ans. The Baſe 538·7 and Perpend. 614·3

2. Given in the Triangle B G D, the Perpendicular G D 76 and the Angle B oppoſite thereto, 56°,,17', quere the Baſe and Hypoth.

Ans. the Baſe 50.72 and Hypoth. 91·37

3. The Baſe X Y 215 and Angle X 41°,,50′, are given in the right angled Triangle X Y Z, to find the Perpendicular and Hypothenuſe.

Ans. The Perpend. 192·5 and Hypoth. 288·6

4. There is a Triangle whoſe Baſe is 318 and Perpendicular 200, required the Angles and Hypothenuſe?

Ans. The ∠ oppoſite the Perpend. 32°,,19′ and Hypoth. 375·7

5. Required the Angles and Perpendicular of the right angled Triangle E F G, when the Hypothenuſe is 70 and Baſe 40.

Ans. The ∠E 55°,,09′ and Per. 57°,,45′.

6. In the Right angled Triangle B O I are given the Baſe 98 and Perpendicular 82, to find the Angles and Hypothenuſe?

Ans. The ∠ B 39°,,38′ and Hypoth. 128·5

VARIETY 3.

VARIETY 3.

Perpendicular Radius.

IF the Perpendicular be made Radius, the Baſe becomes Tangent and Hypothenuſe Secant, ſee Plate 3. Fig. 3. where it is clear (from Def. 12 and 14) that as the Perpendicular is made Radius, the Baſe will be Tangent and Hypothenuſe Secant.

RULE*

As Radius 90
Is to the Perpend.
So is Tangent of the ∠ }
opposite to the Baſe }
To the Baſe.
And
As Radius 90
Is to the Perpend.
So is Secant of the ſame ∠
To the Hypothenuſe.
And the contrary.

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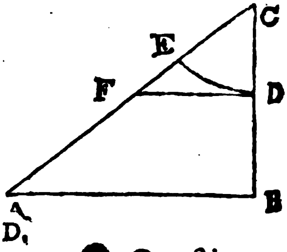
*DEMONSTRATION.

With the Centre C and any Radius C D, deſcribe an Arch D E, and erect the Perpendicular D F which, it is evident, will be the Tangent, and C F the Secant of the Arch D E, or Angle A to the Radius A D. And in ſimilar Triangles C D F, C B A it will be C D : C B : : D F : B A : : C F : C A. Q E D.

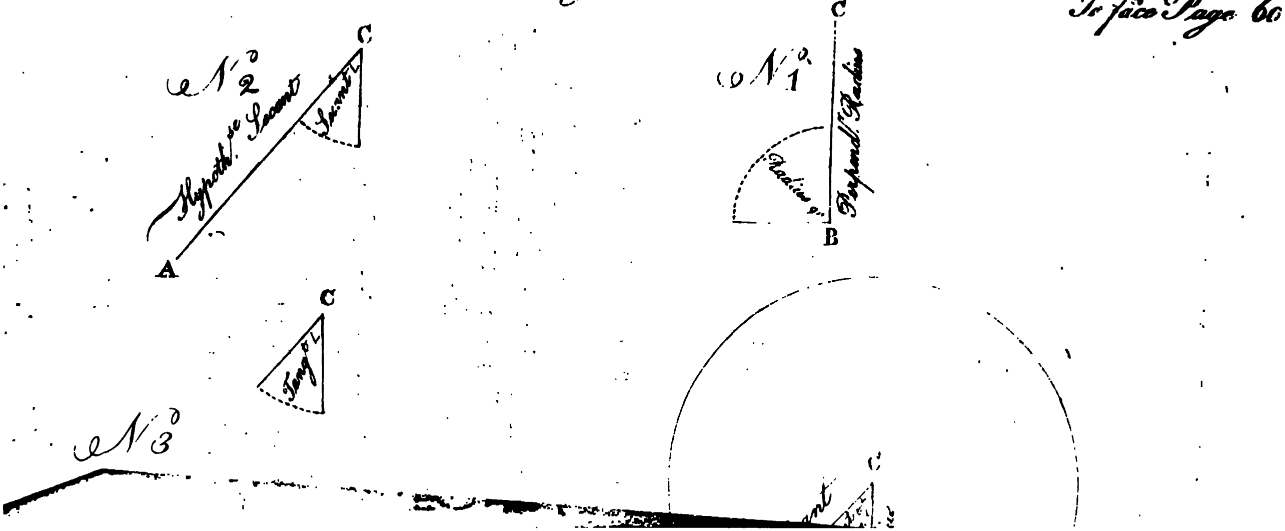


Queſtion 1.

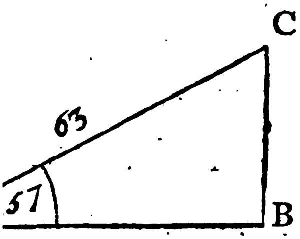
The third Variety Perpend.r Radius.

Plate 3.

To face Page 60



In the right angled Triangle
: given the Hypothenuſe A C
gle A 29°,, 57', to find the
dicular.



THMETICALLY.

enuſe and Angles are here
he Baſe and Perpendicular.
;, and you will ſoon perceive
hat your Proportion will run

the ∠C 60°,, 3'	10·301687
oth. 63	1·799341
f the ∠ C 60°,, 3'	10·239436
54·59	1·737090

And for the Perpendicular, it is made Radius, therefore it muſt be, as ſhewn No. 1, Plate 3. But firſt take either Part of the laſt Proportion.

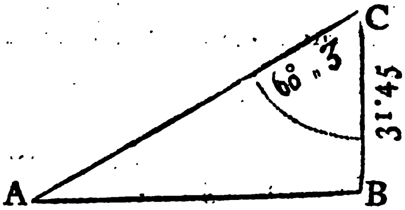
As Secant of the ∠C 60°„30′	10·301687
Is to the Hypoth. 63	1·799341
So is Radius 90	10·000000
To the Perpend. 31·45	1·497654

Or, which will anſwer the ſame Purpoſe.

As Tang. of the ∠C 60°„30′	10·239436
Is to the Baſe 54·59	1·737090
So is Radius 90	10·000000
To the Perpend. 31·45	1·497654

Queſtion 2. In the right angled Triangle A B C, are given the Perpendicular B C 31·45 and Angle A 29°„57′, required the Baſe and Hypothenuſe.

Logarithmetically



LOGARITHMETICALLY.

The Angles and Perpendicular are here given and the Baſe and Hypothenuſe required.

To find the Baſe, it will be,

As Radius 90		10·000000
Is to the Perpend.	31·45	1·497654
So is Tang. of the ∠ C 60°,,30′		10·239436
To the Baſe	54·59	1·737090

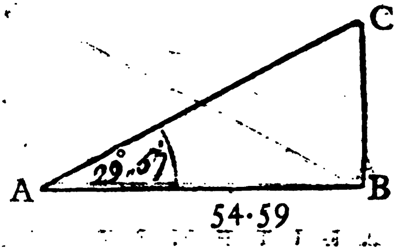
And for the Hypoth.

As Radius 90		10·000000
Is to the Perpend.	31·45	1·497654
So is Secant of the ∠ C 60°,,30′.		10·301687
To the Hypoth.	63	1·799341

or

As Tang. of the ∠C 60°,,30'	10·239436
Is to the Baſe 54·59	1·737090
So is Secant of the ∠C 60°,,30'	10·301687
To the Hypoth. 63	1·799341

Queſtion 3. In the right angled Triangle ABC there are given the Baſe 54·59 and the Angle A 29°,,57', required the Perpendicular and Hypothenuſe?



LOGARITHMETICALLY.

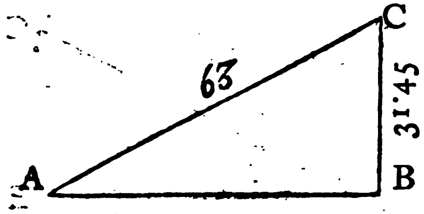
As Tang. of the ∠C 60°,,03'	10·239436
Is to the Baſe AB 54·59	1·737090
So is Secant of the ∠C 60°,,03'	10·301687
To the Hypoth. 63	1·799341

To

To find the Perpendicular.

As Secant of the ∠C 60°,,03′	10·301687
Is to the Hypoth. 63	1·799341
So is Radius 90	10·000000
To the Perpend. 31·45	1·497654

Queſtion 4. In the right angled Triangle ABC, there are given the Perpendicular BC 31·45 and the Hypothenuſe 63 to find the Baſe and Angles.



LOGARITHMETICALLY.

For the Angle C.

As the Perpend. BC 31·45	1·497654
Is to Radius 90	10·000000
So is the Hypoth. AC 63	1.799341
To Secant of the ∠C 60°,,30′	10·301687

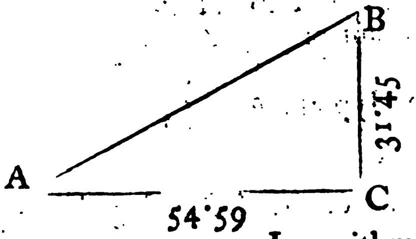
To find the Baſe.

As Radius 90	10·000000
Is to the Perpend. BC 31·45	1·497654
So is Tang. of the ∠C 60° 30′	10·239436
To the Baſe AB 54·59	1·737090

or

As Secant of the ∠C 60° 30′	10·301687
Is to the Hypoth. AC 63	1·799341
So is Tang. of ∠C 60° 30′	10·239436
To the Baſe AB 54·59	1·737090

Queſtion 5. In the right angled Triangle ABC are given the Perpendicular BC 31·45 and Baſe AB 54·59 to find the Hypothenuſe and Angles.



Logarithmetically.

LOGARITHMETICALLY.

To find the Angle C.

As the Perpend. BC 31·45	1·497654
Is to Radius 90	10·000000
So is the Baſe AB 54·59	1·737090
To Tang. of the ∠C 60°,,30'	10.239436

To find the Hypothenuſe.

As Radius 90	10·000000
Is to the Perpend. 31·45	1·497654
So is Secant of the ∠C 60°,,30'	10·301687
To the Hypoth. AC 63	1·799341

or

As Tang. of the ∠C 60°,,30'	10·239436
Is to the Baſe AB 54·59	1·737090
So is Secant of the ∠C 60°,,30'	10·301687
To the Hypoth. AC 63	1·799341

Queſtions to exerciſe the Third Variety.

1. In the right angled Triangle A B C there are given the Hypothenuſe A C 180 and Angle A 45°,,0′ to find the Perpendicular and Baſe.

Ans. The Perpend. 127·3 and Baſe 127·3

2. There is a Triangle the Perpendicular of which is 97 and the Angle oppoſite thereto 29°,,55′, quere the Hypothenuſe and Baſe.

Ans. The Hypoth. 194·5 and Baſe 168·6.

3. Given in the right angled Triangle X O Z, the Baſe X O 58 and the Angle Z 60, to find the Perpendicular and Hypothenuſe.

Ans. The Per. 33·48 and Hypoth. 66·97.

4. Required the Angles and Baſe of the right angled Triangle P O Q, the Hypothenuſe of which is 12 and Perpendicular 8.

Ans. The ∠P 41°,,49′; ∠Q 48°,,11′ and Baſe 8·944.

5. The Perpendicular of the right angled Triangle A O Y is given equal to 75 and Baſe 60, required the Hypothenuſe AY, and Angles.

Ans. Hypoth. A Y 96·06; ∠ A 51° ,, 20′
and ∠ Y 38° ,, 40′.

6. In the right angled Triangle A B C are given the Angle A 34° ,, 36′ and Hypothenuſe A C 120, to find the Baſe and Perpendicular.

Ans. Baſe 98·78 and Perpend. 68·14.

7. Given in the right angled Triangle S T U, the Perpendicular T U 120 and Baſe S T 130, to find the Angles and Hypothenuſe.

Ans. Hypoth. 176·9; ∠ S 42·43 and
U 47° ,, 17′.

Oblique Plain Trigonometry.

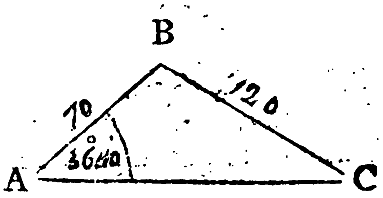
SECTION 4.

IN Definition 5th, it is ſaid, that an oblique angled plain Triangle is ſuch as has no right Angle. As in right angled Triangles there are 6 Caſes, ſo in oblique there are 4, which I ſhall now proceed to explain.

N. B. Oblique angled plain Triangles, as well as right angled ones, have their Sides proportional to each other, as the Sines of their oppoſite Angles, but when there is not a proper Data given for ſuch a Proportion, other Rules are obſerved for bringing out the Solutions, which I ſhall point out in their proper Places.

CASE I.

In the oblique angled plain Triangle A B C, there are given the Side B C 120, the Side A B 70 and the Angle A 36°, 40'; required the other Side and Angles.



Geometrically.

GEOMETRICALLY.

Draw B A and aſſume any Point thereon, as A, with the Chord of 60° in your Compaſſes, and one Foot in A, draw the Arch b a, and ſet off thereon 36°, 40′ through a, draw A B and lay off A B equal 70, then with B C equal 120 in your Compaſſes and one foot in B, croſs with the other, the Line A C, as in C; draw C B, and you will have the Triangle required.

The Angles may be found by taking the Chord of 60°, in your Compaſſes, and with one Foot in C and B deſcribe the Arches a b and c d, which take in your Compaſſes and apply to the Line of Chords, give their reſpective meaſure.

The Side B C being taken in your Compaſſes and applied to a Line of equal Parts gives its Length.

LOGARITHMETICALLY.

As each Side is the Sine of its oppoſite Angle, the Operation will be ſimilar to thoſe in the 1ſt. Variety of right angled Triangles.

As the Side BC 120 2·079181
Is to the Sine of the ∠A 36° 40' 9·776090
So is the Side AB 70 1·845098

To the Sine of the ∠C 20° 24' 9·542007

The Angle C added to the Angle A, and ſubtracted from 180 Degrees, gives the remaining Angle B.

Again; for the Side AC,

As the Sine of the ∠C 20° 24' 9·542007
Is to the Side AB 70 1·845098
So is the Sine of the ∠B 122° 56' 9·923919

To the Side AC 168·7 2·227010

or

As the Sine of the ∠A 36° 40' 9·776090
Is to the Side BC 120 2·079181
So is the Sine of the ∠B 122° 56' 9·923919

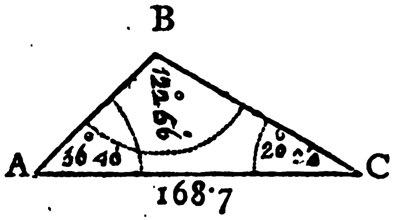
To the Side AC 168·7 2·227010

N. B. In taking the Sine of the obtuſe Angle B, remember that by Definition 7th. the

Sine of 122°„56′ is alſo the Sine of 57°„4′, the one being the Supplement of the other, therefore from the Table of Sines you are to take the Sine of 57°„4′, becauſe every right Sine is the Sine of two Arches, the one being the Supplement of the other.

CASE 2.

In the oblique angled plain Triangle ABC are given, the Angle A 36°„40′, B 122°„56′, C 20°„24′, and the Side AC 168·7, required the Sides AB and BC.



Draw AC, and lay off 168·7 thereon from a Scale of equal Parts.

With the Chord of 60 in your Compaſſes draw the Angle A, and lay off thereon 36°„40′, do the ſame with the Angle C, and lay off 20°„24′, draw AB and BC and the Triangle is completed.

LOGARITHMETICALLY.

To find the Side A B.

As the Sine of the ∠B	122°,,56′	9·923919
Is to the Side A C	168·7	2·227115
So is the Sine of the ∠C	20°,,24′	9·542007
To the Side A B	70	1·845203

Again; to find B C.

As the Sine of the ∠B	122°,,56′	9·923919
Is to the Side A C	168·7	2·227115
So is the Sine of the ∠A	36°,,40′	9·776090
To the Side B C	120	2·079286

or

As the Sine of the ∠C	20°,,24′	9·542293
Is to the Side A B	70	1·845203
So is the Sine of the ∠A	36°,,40′	9·776090
To the Side B C	120	2·079000

CASE 3.

Two Sides and the contained Angle of an *oblique* angled plain Triangle, being given to *find the* other Angles and Side.

RULE*

As the Sum of the two Sides
Is to their Difference
So is the Tang. of half the Sum of the unknown Angles
To Tang. of half their Difference.

To

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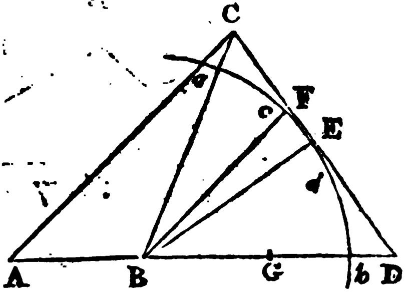
* DEMONSTRATION.



Suppoſe A B C the Triangle, make B D = B C, and B G = A B, then is G D = the Difference of the 2 Sides, and C B D = the Sum of the two unknown Angles (Prop. 5 Sect. 2.) Therefore, let fall the Perpendicular B E, and with that as a Radius, deſcribe the Arch, a E b and draw C D, then will E D be = the Tangent of half the Sum of the two unknown Angles. Draw B F parallel to A C. Then per ſimilar Triangles (Prop. 12 Sect. 5)

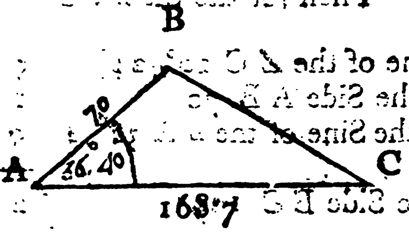
As A G : G D : : D E : E F. Q E D.

But half the Difference ſubtracted from half the Sum gives the leſſer Angle, and added to half the Sum gives the greater.

Suppoſe a E b = the Sum of the two Angles a c the leſs, and c b the greater. Now the Line B E cuts a E b into two equal Parts a E and E b, make E d = c E, then will F H be the whole Difference, and therefore if half the Difference c E be ſubtracted from the half Sum a E, it will give the leſſer a c, and if added c E to E b, gives the greater c b. Q E D.

To half the Sum of the two Angles, add half their Difference, the Sum will be the greater Angle and ſubtracting their half Difference from the half Sum, gives the leſſer Angle.

In the oblique angled plain Triangle A B C, the Side A B 70, A C 168·7 and the Angle between them 36° 40′ are given, to find the other two Angles and Side.



GEOMETRICALLY.

Draw A B and ſet off 70 from a Scale of equal Parts.

With the Chord of 60 in your Compaſſes deſcribe the Angle A and ſet off thereon 36° 40′. Draw A C equal 168·7 and join C B; which completes the Triangle.

Logarithmetically.

ARITHMETICALLY.

As the Sum of two Sides 238·7	2·377852
Is to their Difference 98·7	1·994317
So is the Tang. of half the Sum of the unknown ∠s 71°,,40'	10·479695
To Tang. of half their Difference 51°,,18'	10·096160

Then for the Side B C.

As Sine of the ∠ C 20°,,24'	9·542007
Is to the Side A B 70	1·845098
So is the Sine of the ∠ A 36°,,40'	9·776090
To the Side B C 120	2·079181

CASE 4.

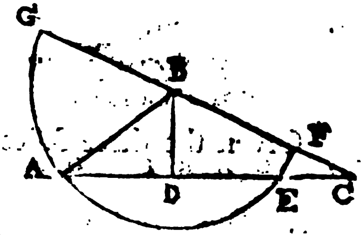
Given the three Sides to find the Angles.

RULE.

As the Baſe or largeſt Side
Is to the Sum of the other two Sides
So is the Difference of thoſe Sides
To the Difference of the Segments of the Baſe, made by a Perpendicular let fall from the vertical Angle upon the ſaid Baſe. Then half the Difference added to half the longer Side, gives the greater Segment, and ſubtracted from it, gives the leſſer.

In

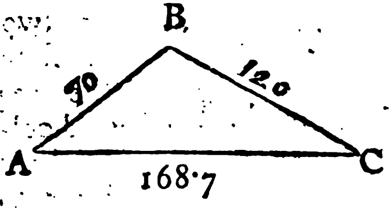
DEMONSTRATION.



From one end B of the leaſt Side A B of the Triangle A B C as a Centre, and Radius B A, deſcribe a Circle cutting the other two Sides in E and F; and let fall the Perpendicular B D. Then it is evident that $GB = BF = AB$, and by (Prop. 13. Sect. 2) $AD = DE$, and conſequently $EC = CD - DA$, $FC = CB - BA$, and $GC = CB + BA$. But (Prop. 9 Sect. 2.) $AC : CG :: FC : CE$ that, is, $AC : CB + BA :: CB - BA : CD - DA$. Q. E. D.

And that the half Sum of two Quantities increaſed and deminiſhed by their half Difference, gives the greater and leſs Quantities reſpectively, was proved in the laſt Caſe.

In the oblique angled plain Triangle ABC are given AB 120, AC 168·7 and BC 70, to find the Angles A, B and C.



GEOMETRICALLY.

Draw the Line AC and lay off thereon 168·7 from a Scale of equal Parts. From the ſame Scale take 70 in your Compaſſes and one Foot in A deſcribe an Arch, then take 120 and with one Leg in C deſcribe with the other, as ſecond Arch cutting the Former in B, which completes the Triangle.

The Angles may be meaſured as before, i, e, from a Line of Chords.

LOGARITHMETICALLY.

As Baſe 168·7	2·227115
Is to the Sum of the other two Sides 190	2·278754
So is the Diff. of thoſe Sides 50	1·698970
To the Diff. of the Seg. of the Baſe 56·31	1·758609

Now half the Difference of the Segments is 28·15 which added to 84·35 = half the longer Side, gives the greater Segment, and ſubtracted from it gives the leſſer; therefore the Angle B may be found as follows.

As the Hypoth. AB 70	1·845098
Is to Radius 90	10·000000
So is the Baſe AD 56·2	1·749736
To the Sine of the ∠B 53°.24′	9.904638

The Angle B ſubtracted from 90 Degrees leaves the ∠A, or it may be found by making the Baſe Radius, thus:

As the Baſe AD 56·2	1·749736
Is to Radius 90	10·000000
So is the Hypoth. AB 70	1·845098
To the Secant of the ∠A	10·095262

Queſtions

Questions to exercise the Four Cases of oblique angled plain Trigonometry.

1. In the oblique angled plain Triangle ABC are given the Side AB 86, the Side BC 137 and the Angle C 42°,,10', required the Side AC and the other Angles.

Ans. The Side AC 187·2; the ∠A 40°,,15' and ∠B 117°,,35'.

2. Given in the oblique angled plain Triangle DGH, the ∠D 27°,,40', the Angle G 107°,,26', Angle H 44°,,54'; and the Side DH 120, required the other two Sides.

Ans. GH 58·4 and the Side DG 88·78.

3. There is an oblique angled plain Triangle XOY, the Side XO 410, XY 600 and the Angle between them 38°,,50', required the other Angles and Sides.

Ans. ∠Y 42°,,30'; ∠O 98°,,40' and the Side YO 381.

4. In the oblique angled plain Triangle A B C are given the Side A B 110, B C 80 and A C 150, required the Angles.

Ans. The ∠ A 31°„17′; ∠ C 45°„35′;
∠ B 103°„08′.

5. Given in the oblique angled plain Triangle C D E the Side C D 127, C E 200 and the ∠ D 109°„0′, required the other Side and Angles.

Ans. The Side D E 118·5; ∠ C = 34°„6′
and ∠ E 36°„54′.

6. The Side A B 69, and A C 90, and the Angle between them 50°„45′ are given in the oblique angled Triangle A B C, to find the other Side and Angles.

Ans. The Side B C 70·73; ∠ B 80°„10′
and ∠ C 49°„4′.

Heights and Diſtances.

CHAP. 2.

SECTION 5.

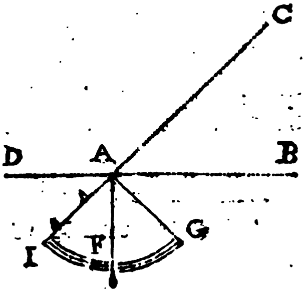
THE Object to be meaſured may be either acceſſible or inacceſſible.

Acceſſible Lines are meaſured by the Application of ſome certain Meaſure to them, as a Foot, a Yard, a Chain &c. a certain Number of Times.

Inacceſſible Lines are meaſured by Means of Angles: Angles are taken and then the Inacceſſible Lines are found by Trigonometrical Calculations.

To take an Angle of Altitude with the Quadrant.

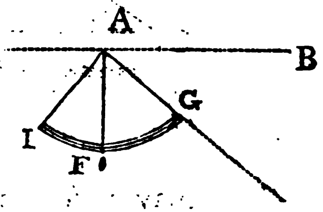
Suppoſe C to be any Object as the Top of a Mountain, Tree or Houſe, and you want to know what Angle it makes with the Horizon from the Point A, Firſt fix the Centre of your Quadrant at A, then move it up and down till ſuch Times as you obſerve through the Sights the Object C, then



then obſerve what Number of Degrees F G is cut by the Plum A F, and that will be the Meaſure of the Angle C A B as required. The truth of this may eaſily be demonſtrated, thus:

Firſt produce B A to D, then is D A F equal to a right Angle (by Prop. 1. Sect. 2.) and I F G is known to be equal to a right Angle. But D A I + I A F = F A G + I A F. Therefore F A G = D A I (becauſe I A F is common to both) which alſo is = C A B (Prop. 2. Sect. 2.) Q E D.

To take an Angle of Depreſſion.

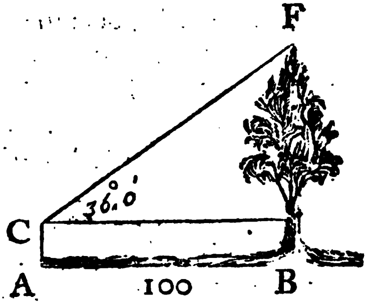


This is done in the manner as the laſt, only here you muſt apply the Centre of the Quadrant to the Eye, as per Figure, and the Arch F G is the Angle required.

DEMONSTRATION.

The Angle G A I which the Inſtrument makes, is equal to 90° and B A F is equal to 90° (Prop. 1. Sect. 2.) But G A F + F A I = B A I + F A I, therefore G A F = B A F. Q E D.

Wanting to know the Height of a large Elm Tree, which ſtood upon a horizontal Plain, I meaſured from its bottom B in a direct Line juſt 100 Yards to A, there I took the Angle of Elevation of the Top, which I found juſt 36°,0′, I demand the Height of the ſaid Tree, the Quadrant being four Feet from the Ground.



This may be ſolved by the 1ſt. Variety.

As the Sine of the ∠ F 54	9·907958
Is to the Baſe A B 100	2·000000
So is the Sine of the ∠ C 36°	9·769219
To the Tree's Altitude 72·65	1·861261

To which add four Feet the Height of the Inſtrument, and it gives the true Altitude 76·65 Yards.

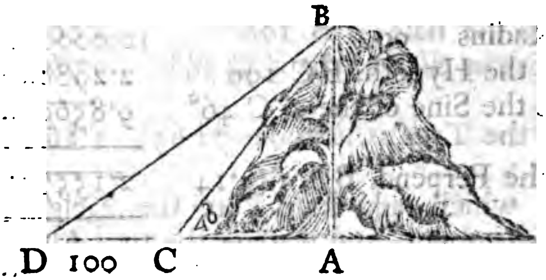
Or, thus by making the Baſe Radius.

As Radius 90	10·000000
Is to the Baſe A B 100	2·000000
So is the Tang. of the ∠ C 36°	9·861261
To the Tree's Altitude 72·65	1·861261

Or, thus by making the Perpendicular Radius.

As Tang. of the ∠ F 54°	10·138739
Is to the Baſe 100	2·000000
So is Radius 90	10·000000
To the Tree's Altitude 72·65	1·861261

What is the Perpendicular Height of a Hill, whoſe Angle of Elevation taken at the Bottom of it was 46°, and 100 Yards farther off on a Level with the Bottom of it, was 31°.



The external Angle BCA made leſs by the Angle D is equal to the Angle DBC (per Prop. 5. Sect. 2.)

46° } Subtract.
31°

15°

By Caſe 1. of oblique Trigonometry.

As the Sine of the ∠ DBC 15°	9·412996
Is to DC 100	2·000000
So is the Sine of the ∠ BDC 31°	9·711839
To the Side BC 199	2·298843

Then to find BA the Height of the Hill, proceed by the 1ſt. Variety of right angled Triangles.

As Radius 90	10·000000
Is to the Hypoth. BC 199	2·298843
So is the Sine of the ∠ C 46°	9·856934
To the Perpend. BA 143·14	2·155777

It may alſo be found by the 2d. and 3d. Varieties, thus:

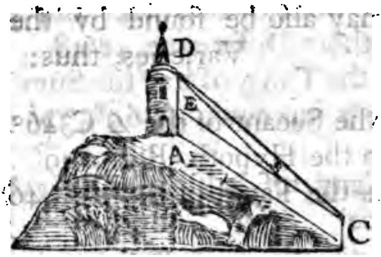
As the Secant of the ∠ C 46°	10·158229
Is to the Hypoth. B C 199	2·298843
So is the Tang. of the ∠ C 46°	10·015163
To the Perpend. B A 143·14	2·155777

and

As the Secant of the ∠ B 44°	10·143066
Is to the Hypoth. 199	2·298843
So is Radius 90	10·000000
To the Perpend. 143·14	2·155777

An Obeliſk ſtanding on the Top of a Declivity, I meaſured from its Bottom a Diſtance of 40 Feet, and then took the Angle formed by the Plain and a Line drawn to the Top 41°; and going on in the ſame Direction 60 Feet farther, the Angle there was 23°, 45′ the Height of the Inſtrument being 5 Feet. What was the Height of the Obeliſk.

You



You muſt firſt find the Side B D which you may do by Caſe 1. of oblique angled Trigonometry.

As Sine of the ∠ D 17° 15′ 9·472086
Is to B C 60 1·778151
So is the Sine of the ∠ C 23° 45′ 9·605032

To the Side B D 81·48 1·911097

Now in the Triangle E B D you have two Sides E B = 40, B D = 81·48 and the Angle contained between them 41° to find the Side D E.

In the firſt place work for the other two Angles by Caſe 3d. of oblique Trigonometry, thus:

As the Sum of two Sides 121·48	2·084219
Is to their Difference 41·48	1·617839
So is the Tang. of half the Sum of the 2 unknown ∠s 69°,,30′	10·427262
To Tang. of half their Difference 42°,,25′	9·960882
111,,55 Greater ∠ E	
27,,5 Leſſer ∠ D	

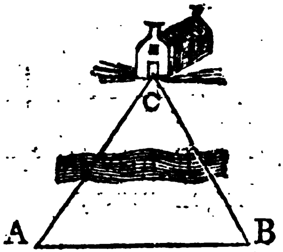
Then the Side DE may be found by Caſe 1ſt. of oblique Trigonometry.

As the Sine of the ∠ E 111,,55	9·967420
Is to the Side B D 81·48	1·911097
So is the Sine of the ∠ B 41°	9·816943
To the Side D E 57·63	1·760620

AE 5

AD=62·63 the Height required.

Being on the Side of a River and wanting to know the Diſtance of a Houſe which ſtood on the other Side, I meaſured 200 Yards in a right Line by the Side of the River and found the two Angles at each End of this Line formed by the other End and the Houſe were A = 73°,,15′ and B = 68°,,2′. What was the Diſtance between each Station and the Houſe?



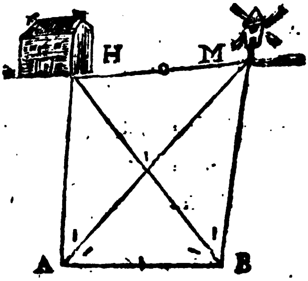
For the Side BC, by Caſe 1ſt. oblique Trigonometry.

As the Sine of the ∠ C	38°,,43′	9·796206
Is to the Side AB	200	2·301030
So is the Sine of the ∠ A	73°,,15′	9·981171
To the Side BC	306·19	2·485995

For the Side A C, by the ſame.

As Sine of the ∠ C 38°,,43′	9·796206
Is to the Side A B 200	2·301030
So is the Sine of the ∠ B 68°,,2′	9·967268
To the Side A C 296·54	2·472092

Wanting to know the Diſtance between a Houſe and a Mill which were ſeperated from me by a River, I took another Station B, at the Diſtance of 300 Yards from the firſt Station A, the Angle ſubtended by B, and the Mill was 58°,,20′, and by the Mill and the Houſe 37°; from B, the Angle ſubtended by A and the Houſe was 53°,,30′, and by the Houſe and Mill 45°,,15′. What is the Diſtance of the Houſe and Mill?



In the Triangle A B H, you have all the Angles given, for the Angle H B A=53°,,30′ and the Angle M A B and H A M = 58°,,20′ and 37°,,00′, reſpectively, which are equal 95°,,20′ = the ∠ H A B, therefore the Angle H A B added to the Angle H B A and ſubtracted from 180 Degrees will give the remaining Angle A H B, thus:

95,,20
53,,30

148,,50
180,, 0

A H B = 31,,10

And the Angle H B A + H B A gives the Angle A B M = 98°,,45′ which added to the Angle A and ſubtracted from 180° gives the remaining Angle, thus:

A B M 98°,,45′
A 58,,20

157,,5
180·0

∠ A M B 22,,55

Then for the Line A H.

As the Sine of the ∠ A H D	31°,,10'	9·713935
Is to the Side A B	300	2·477121
So is the Sine of the ∠ABH	53°,,30'	9·905179
To the Side A H	465·91	2·668365

For the Side A M, by the ſame.

As the Sine of the ∠ AMB	22°,,25'	9·590386
Is to A B	300	2·477121
So is the Sine of the ∠ABM	98°,,45'	9·994915
To the Side A M	761·46	2·881650

Now in the Triangle A H M you have two Sides and the ∠ between them, to find the other Side and Angles, you muſt therefore proceed by Caſe 3 of oblique Trigonometry.

	761·46		180°,,0'	
	465·91		37,, 0	∠M A H
The Sum	1227·37	of 2 S.	143,, 0	ſum of 2 ∠s
Their	295·55	Diff.	71,, 30	Half Do.

As the Sum of 2 Sides 1227·37 3·088890
Is to the Difference 295·55 2·460631
So is Tang. of half the Sum of the 2 unknown ∠s 71°„30′ 10·475480

To Tang. of half their Diff. 35„7 9·847221

106„37 Greater ∠ H

36„21 Leſſer ∠ M

Next for the required Diſtance.

As the Sine of the ∠ AMH 36°„21′ 9·772847
Is to the Side A H 465·9 2·668365
So is the Sine of the ∠ HAM 37„0′ 9·779463

To the Side HM 474·2 2·675981

PROMISCUOUS

Promiſcuous Queſtions.

1. IN the right angled Triangle A B C there are given the Hypothenuſe A C 100, the Angle A 33°„45′ to find the Perpendicular and Baſe.

Ans. Perpend. 55·56 and Baſe 83·15.

2. In the oblique angled plain Triangle C D E the Side C D = 50, C E = 80 and the Angle D C E which is contained between them = 38°, quere the other Side and Angles.

Ans. The ∠E 37°„10′, ∠D 104°„50′
and the Side 50·59.

3. There is a Triangle A B C whoſe Baſe A B is 80 and Perpendicular B C 100, what are the Angles and Hypothenuſe?

Ans. The ∠A 51°„20′, C 38°„40′ and
Hypoth. 128.

4. Wanting to know the Breadth of a River, I meaſured 150 Yards in a ſtraight Line cloſe by one Side of it, and at each End of this Line I found the Angles ſubtended by the other End, and a Tree cloſe by the other Side of the River, to be 63° and 75°,,30′. What is the Perpendicular Breadth.

Ans. 130·2 Yards.

5. There is an oblique angled plain Triangle whoſe three Sides AB, BC and CA are 60, 100 and 120 reſpectively, required the Angles.

Ans. ∠C 29°,,56′, ∠B 93°,,49′ and ∠A 56°,,15′.

6. In the right angled plain Triangle DEF are given DE the Baſe 76, and Hypothenuſe DF 90, quere the Angles and Perpendicular.

Ans. ∠F 57°,,37′, ∠D 32°,,23′, and Perpendicular 48·09.

7. In the oblique angled plain Triangle A B C, are given the ∠ A 37°, B 138° and C 25° and the Side A B 109; required the other two Sides.

Ans. The Side B C 117·1 and A C 172·9.

8. From a Ship at Sea I obſerved a Point of Land to bear E by S, and after ſailing N E 12 Miles I ſet out again and found its bearing to be S E by E. How far was the laſt Obſervation made from the Point of Land.

Ans. 26·07 Miles.

9. Given in the right angled Triangle, A B C the Hypothenuſe A C 650 and Perpendicular B C 500, I demand the Angles and Baſe.

Ans. The ∠ A 50°,, 17′, ∠ C 39°,, 43′ and
Baſe A B 415·4.

10. Wanting to know the Height of a Summer Houſe, which ſtood upon a level Plat of Ground, I meaſured 60 Yards from its Bottom

in a direct Line, and there took the Angle of Elevation of the Top, which I found to be 52°,,0', and going 80 Yards farther, found the Angle there only 28° ; quere the Height of the ſaid Summer Houſe the Quadrant being placed 6 Feet above the Ground ?

Ans. 76·76 Yards.

11. There is a Triangle A B C, whoſe Hypothenuſe A C is 240, and Baſe A B 220; what are the Angles.

Ans. The ∠ A 36°,,2', and ∠ C 53°,,58'.

12. From the Top of a Ship's Maſt, which was 80 Feet above the Water, the Angle of Depreſſion of another Ship's Hull upon the Water at a Diſtance is 20°; what is their Diſtance.

Ans. 219.79 Feet.

FINIS.

ERRATA.

Page 14 Line 5 for unequal, read equal
Do. Line 7 for unequal, read equal.
Page 36 Line 2 for A B read C B.
Page 38 Line 16 for No. 3 read No. 2.
Page 45 Line 15 for No. 1 read No. 3.
Page 83 *Dele* Section 5.